Dynamic Modeling of Overcurrent Protection Relays for Open-Loop Response to Electrical Phasors in Power Systems

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Abstract

This research developed analytical models of overcurrent protection relays for the simulation of their response to electrical transients in power systems. The dynamic models of the relays were based on the open-loop characterization test results of several relays conducted in previous research. The accuracy of the models were verified by comparing the simulated phase current output of the model with empirical test data and the relay manufacturers' published characteristic curves.

The mathematical models of the overcurrent relays are an improvement over the manufacturers' published time-current characteristic very inverse curves, which are based on steady-state testing under normal power frequency conditions. Time overcurrent surface plots of operating time with respect to multiples of pickup current and frequency reveal frequency ranges where relay operation is the same as in 60 Hz, with an error of less then 1.5 cycles. Color-coded density diagrams of operating time against frequency and current reveal optimal regions for instantaneous mode operation for the relays modeled.

I. Introduction

Detailed modeling of the power system is becoming more and more a necessity for utilities embarking in any form of asset expansion. Modeling checks proposed systems for potential problems such as mal-operation and incompatibility.

Protection relays detect faults in the power system and issue tripping signals to isolate compromised portions of the network. The first generation relays are electromechanical, the second generation are solid state. The third and current generation is microprocessor-based. Protection relays are mainly classified into overcurrent relays, distance relays, and differential relays. The most common of these are overcurrent relays. They are being used as primary and backup protective devices and can be found in every type of power system protection zone. It is the primary protection for distribution feeders. It is also used as backup protection for transmission lines against phase or ground faults [1].

Manufacturers give overcurrent relay data in the form of time current characteristic curves (TCC). These come from the relay design that assumes a phasor input such as 60 Hz. These cannot predict relay operation in the case of evolving currents and in the presence harmonics or off-nominal frequency currents. A transient model can predict relay operation for evolving currents, distorted currents, and off-nominal frequency currents.

Current modeling methodologies are based on relay construction and operation [2] [3]. Modeling may also be based on the results of relay characterization, wherein a set of inputs is fed to the relay and the outputs are recorded and analyzed in order to develop mathematical models that fit the data. The advantages are many. Characterization-based methodologies do not differentiate between relay generations [4]. The process does not require detailed relay construction information. The results are approximate but accurate for the range where there is sufficient data, that is, the range characterized.

II. Methodology

Input waveforms may be classified into two, the phasor or the single frequency wave, and the mixed frequency waveform. It is the phasor The Relay Model was developed for each relay supports one to two modes (device 50 or device 51), depending on if the mode exists in the relay. The research methodology developed analytical models for each relay under study, for each of its supported modes, for each of the input waveforms that was characterized in previous research. An expanded flowchart is shown in Figure 1.

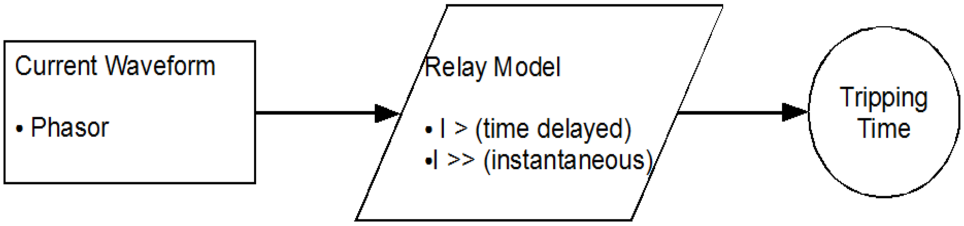

Figure 1: Expanded flowchart

2.1 Model Development

The relay model in time-delay mode for a phasor input takes the wave amplitude and the frequency of the wave to determine if the relay will operate and the time it will take to do so if it does. This is illustrated in Figure 2. The model was created through regression analysis. The basis function is the form of the IEEE very inverse curve equation, which is shown in (1).

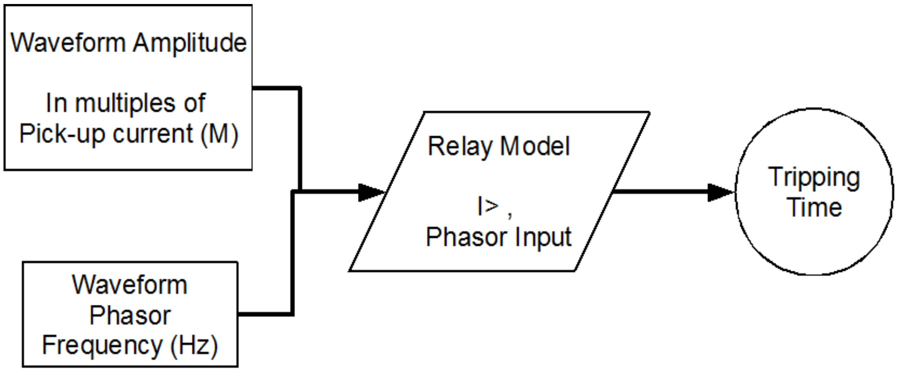

Figure 2: Model operation for 'I >' mode and phasor input

$$t_{trip}(M)=TD\left(\frac{A}{M^{P}-1}+B\right)+K \tag{1}$$

The relay model in instantaneous mode for a phasor input takes the wave amplitude and the frequency of the wave to determine if the relay will operate and the time it will take to do so if it does. This is illustrated in Figure 3. The model was created through regression analysis. The basis function will be a high-degree polynomial because it has been found in literature that the operating time for this may have very arbitrary waveforms. An example for this basis function is shown in (2).

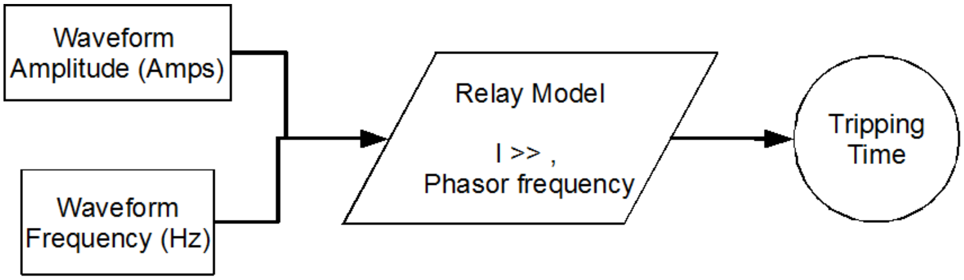

Figure 3: Model operation for 'I >>' mode and phasor input

$$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 \tag{2}$$

2.2 Validation

The principle of relay compliance states that the model must agree with experimental data on whether the relay trips or does not trip. It must also agree with the time that the relay takes to operate. To validate the models that will be created, the output of the model were compared with the data from the manufacturers as well as from previous research. Graphs were plotted to show both manufacturer's curves and the output of the model. Data points from field experiments in previous research will be included as well as the difference between the model and known data. The root mean square error (RMSE) was computed for the models created.

III. Results and Discussion

Transient modeling based on relay characterization begins with recreating the transient input signals as well as the plotting the performance of the relay against the input given. The mathematical model is then developed to achieve close agreement with the empirical data for each of the relays characterized. The list of relays and corresponding descriptions are in Appendix A.

3.1 Time Overcurrent Relay Models

The phasor waveforms are in the form $A\sin(2\pi ft)$. A is the amplitude of the sinusoid. The frequency in Hertz is f. Previous research has experimented on with a frequency range of 10 to 300 Hz. This covers the frequencies around 60 Hz as well as the second to the fifth harmonics. The time in seconds is t. For the purpose of plotting, we have used a time step of $(0.01 \div f)$.

The relays characterized were set to assume very inverse characteristic curves. IEEE standard curves take the form of (3). D is the time dial setting, M is the multiple of pickup current. The rest are constants. This form is general enough to model non-IEEE characteristic curves such as those used by IEC.

$$t_{trip}(I)=D\left(\frac{A}{M^{P}-1}+B\right)+K \tag{3}$$

This equation is used to describe the operating time of the relay set to very inverse (VI) for any value of M for a given frequency.

Data fitting is done to solve the values of D, A, B, p, and K. The software EzyFit was used to do this. Ezyfit is a free third-party software add on to MATLAB.

Figure 4 shows a typical example of the models produced. The value of the constants for the mathematical model are also shown. R describes the residue. The RMSE is approximately 1.5 cycles long, which is acceptable for relay coordination purposes which normally have an allowance of 0.3 seconds or more (18 cycles) for electromechanical relays.

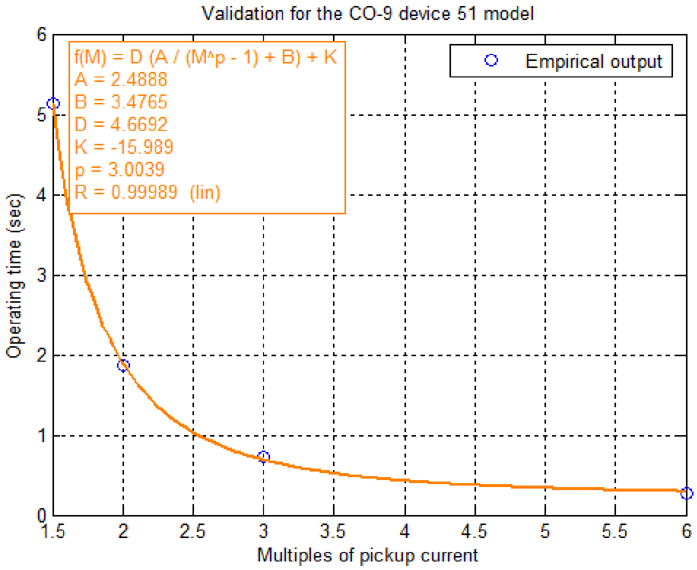

Figure 4: Data fit for CO-9 VI time overcurrent operation

An equation was developed for each of the frequencies characterized for the C0-9 relay. Table 1 gives the values for the constants to describe the time overcurrent operation for the frequencies characterized for the CO-9 relay as well as the corresponding error estimates. The same was done for the G.E. 12IAC53A1A electromechanical relay in Table 2.

Table 1: Model constants for the CO-9 relay

Freq (Hz)	40	50	60	70	80	90	100	110
A	2.49	1.42	1.40	1.64	1.83	3.02	2.99	3.80
B	3.48	-13.27	-0.39	0.40	0.89	-0.60	-1.33	-0.15
D	4.67	2.67	3.15	2.30	2.26	1.69	2.17	2.43
K	-15.99	35.47	1.34	-0.85	-1.92	1.12	2.99	0.51
p	3.00	1.79	2.21	2.06	2.16	2.35	2.54	2.84
RMSE	2.85E-02	5.57E-02	7.93E-04	3.40E-03	4.07E-03	9.14E-04	4.41E-03	1.14E-02

Table 2: Model constants for the IAC53 relay

Freq (Hz)	30	40	50	60	70	80	90	100	110	120	180.00	240	300
A	6.61	2.30	1.74	1.77	2.53	2.11	1.67	2.02	1.75	1.88	1.72	1.90	1.18
B	-6.04	-0.80	0.74	0.49	0.68	1.01	0.11	0.67	-0.24	0.00	0.38	0.56	0.14
D	2.33	2.84	2.58	2.07	1.31	1.46	1.77	1.47	1.78	1.69	2.22	2.59	5.45
K	14.37	2.47	-1.77	-0.89	-0.79	-1.39	-0.11	-0.92	0.51	0.08	-0.76	-1.34	-0.65
p	3.21	2.45	2.15	2.00	1.92	1.86	1.82	1.83	1.87	1.89	2.03	2.23	2.44
RMSE	4.26E-02	1.07E-02	1.28E-03	2.02E-03	1.59E-03	1.08E-03	3.98E-03	3.80E-03	3.87E-03	4.25E-03	1.36E-03	3.59E-03	9.42E-03

Very inverse curves such as that in Figure 4 look similar to each other. For the electromechanical relays, the operating time is of the range 0.2 to 6 seconds depending on the multiple of pickup current which has been characterized for 1.5 to 6 multiples. The average RMS error for the CO-9 relay and IAC53 relay models, with their corresponding standard deviations are shown in Table 3.

Table 3: Error statistics (E-M)

RMSE	CO-9	IAC53
Mean	1.3651E-02	6.8908E-03
S. Dev.	1.9287E-02	1.1141E-02

For the solid state relay models, the RMSE is typically in the single-digit millisecond range. Table 4 gives the values for the constants to describe the time overcurrent operation for the frequencies characterized for the BE1B relay as well as the corresponding error estimates. The same was done for the G.E. DIACA5A solid state relay in Table 5 and for Siemens 7SK8854 in Table 6.

Table 4: Model constants for the BE1B relay

Freq (Hz)	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**180**	**240**	**300**
A	0.96	0.94	7.58	2.47	2.30	4.64	0.29	1.75	1.09	1.14	0.73	0.78	1.48	0.74
B	0.54	0.49	-1.38	-2.11	0.03	-2.72	0.76	2.02	0.50	0.63	0.66	0.36	0.36	0.08
D	2.89	2.94	0.38	1.05	1.16	0.57	9.02	1.43	2.30	2.24	3.24	3.23	1.66	3.17
K	-1.46	-1.35	0.62	2.30	0.05	1.64	-6.73	-2.82	-1.07	-1.32	-2.07	-1.08	-0.52	-0.19
p	2.19	2.15	2.20	2.08	2.12	2.12	2.09	2.04	2.05	2.07	1.97	2.07	2.05	2.00
RMSE	9.49E-03	7.04E-03	5.73E-03	2.20E-03	3.52E-03	3.34E-03	2.97E-03	4.05E-03	2.27E-03	8.32E-04	2.00E-04	3.61E-03	1.07E-02	9.07E-04

Table 5: Model constants for the DIAC relay

Freq (Hz)	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**
A	5.04	0.63	1.92	1.03	1.99	1.48	1.71	1.71
B	-2.98	0.15	-0.58	1.38	-0.36	0.39	0.05	0.05
D	0.49	3.08	1.01	1.94	0.94	1.30	1.17	1.17
K	1.55	-0.38	0.66	-2.60	0.42	-0.43	0.02	0.02
p	2.24	1.97	2.01	2.04	1.98	2.01	2.05	2.05
RMSE	9.18E-04	1.33E-03	3.42E-03	1.40E-04	1.15E-03	2.25E-03	1.16E-03	2.56E-03

Table 6: Model constants for the 7SK88 relay

Freq (Hz)	20	30	40	50	60	70	80	90	100	110	120	180	240
A	2.44	1.70	1.77	2.21	2.07	4.28	2.33	1.89	1.06	2.02	1.81	1.81	2.08
B	3.64	-0.26	-1.99	-3.75	-0.78	0.41	2.91	0.76	0.23	0.80	-1.32	-1.32	1.08
D	0.99	1.46	1.37	1.10	1.18	0.55	1.04	1.28	2.27	1.20	1.33	1.33	1.13
K	-3.60	0.39	2.72	4.15	0.93	-0.22	-3.02	-0.96	-0.51	-0.96	1.77	1.77	-1.21
p	2.00	2.05	2.02	2.03	2.03	1.99	2.03	2.02	2.02	2.03	2.02	2.02	1.99
RMSE	8.77E-03	7.79E-03	7.10E-03	9.84E-03	9.78E-03	9.26E-03	5.33E-03	5.61E-03	5.62E-03	5.50E-03	5.59E-03	5.59E-03	4.97E-03

For the static analog relays, the operating time is of the range 0.2 to 2 seconds depending on the multiple of pickup current which has been characterized for 1.5 to 6 multiples. The average RMS error for the solid state relay models, with their corresponding standard deviations are shown in Table 7.

Table 7: Error statistics (solid state)

RMSE	BE1B	DIAC	7SK88
Mean	4.0634E-03	1.6161E-03	6.9818E-03
S. Dev.	3.1587E-03	1.0489E-03	1.8623E-03

An mathematical model was developed for each of the frequencies characterized for the digital relays. Table 8 gives the values for the constants to describe the time overcurrent operation for the frequencies characterized for the BE1 relay as well as the corresponding error estimates. The same was done for the ABB SPAJ140C digital relay in Table 9, for G.E. Multidia SR-760 relay in Table 10, and for Schweitzer SEL-587 relay in Table 11.

Table 8: Model constants for the BE1 relay

Freq (Hz)	40	50	60	70	80
A	2.20	3.12	0.70	2.80	2.96
B	0.00	2.20	0.72	1.46	0.51
D	6.19	1.22	3.93	1.34	4.57
K	0.23	-2.57	-2.72	-1.85	-2.12
p	3.45	2.28	2.04	2.27	3.41
RMSE	4.27E-02	1.97E-03	7.67E-03	7.67E-03	3.75E-02

Table 9: Model constants for the SPAJ140C relay

Freq (Hz)	30	40	50	60	70	80	90
A	0.06	0.06	0.74	0.35	1.51	7.57	3.69
B	1.07	1.21	1.13	-0.02	-0.50	-2.14	-2.42
D	2.68	2.60	0.71	3.05	0.86	0.35	5.48
K	-2.93	-3.21	-0.78	0.15	0.52	0.89	13.56
p	0.31	0.29	0.84	1.43	1.54	2.15	4.40
RMSE	3.07E-03	4.10E-03	1.43E-03	2.81E-03	7.41E-03	1.73E-02	6.33E-02

Table 10: Model constants for the SR-760 relay

Freq (Hz)	**50**	**60**	**70**
A	2.22	0.99	3.96
B	-1.44	-0.27	-6.15
D	1.09	1.37	0.59
K	1.61	0.40	3.66
p	2.01	1.66	1.96
RMSE	1.17E-02	9.38E-03	1.49E-02

Table 11: Model constants for the SEL-587 relay

Freq (Hz)	**40**	**50**	**60**	**70**	**80**	**90**
A	32.29	1.37	1.41	0.99	1.98	5.53
B	1.77	0.79	0.40	-0.27	-0.46	-3.99
D	3.71	4.97	1.30	1.37	1.45	2.15
K	-6.22	-3.80	-0.47	0.40	0.75	8.73
p	5.76	3.07	1.93	1.66	2.35	3.37
RMSE	1.17E-01	2.18E-02	3.97E-04	6.72E-03	6.83E-03	3.04E-02

For the static digital relays, the operating time is of the range 0.2 to 5 seconds depending on the multiple of pickup current which has been characterized for 1.5 to 6 multiples. The range of operating times widely varies with each relay. The average RMS error for the solid state relay models, with their corresponding standard deviations are shown in Table 12.

Table 12: Error statistics (digital)

RMSE	**BE1**	**SPAJ**	**SR**	**SEL**
Mean	1.9502E-02	1.4201E-02	1.2005E-02	3.0591E-02
S. Dev.	1.9034E-02	2.2316E-02	2.7937E-03	4.3949E-02

Newer generation relays tend to need less allowance. Typically 0.1 to 0.2 seconds or 6-12 cycles are given. The RMSE for the models of these relays are correspondingly smaller. The average RMSE for the time overcurrent models is 8.85 milliseconds with a standard deviation of 6.52 milliseconds.

Aside from validation against experimental data coming from previous research, validation against the manufacturer's published design specifications was done. Manufacturer's publish the performance models of their relays for a phasor input of 60 Hz. This was compared to the developed models for the same input. The blue curve in Figure 5 comes from the manufacturer's manual. The green curve comes from the model developed and the black curve at the bottom is the absolute difference between the previous two.

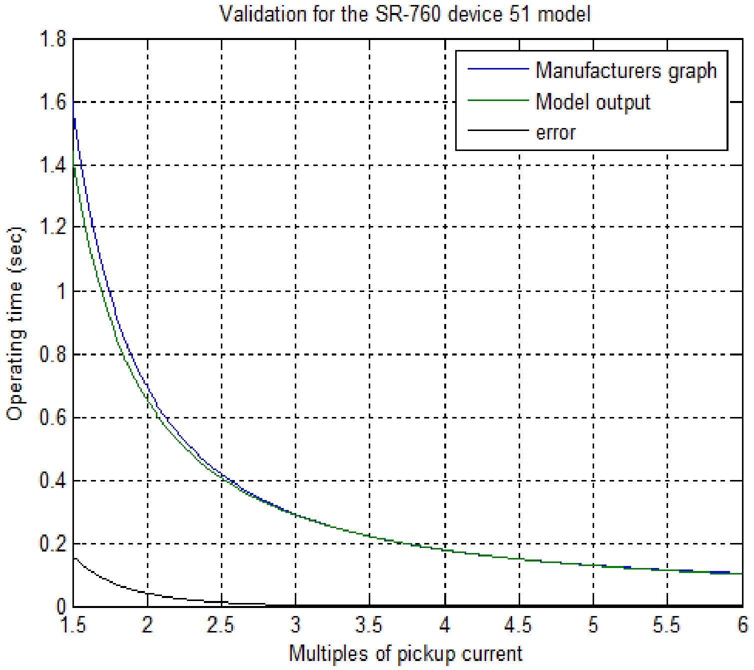

Figure 5: SR model validation against the manufacturer's curve

The error values for the electromechanical relays, the solid state relays and the microprocessor-based relays are listed in Table 13, Table 14, and Table 15, respectively. The error for relay models do not show significant differentiation among relay generations. The RMSE of the models are all more or less of the same magnitude. The maximum error and the RMS percent error were computed to predict the conditions where the model is more accurate. If the relay is expected to experience inputs with low multiple of pickup (M) values, having a model with a small maximum error will provide accurate results even with higher RMS percent errors. However, when the relay is expected to experience inputs with high values of M, having a low RMS percent error will provide accurate results, even with higher maximum error values. The model errors are distributed in three ways. They can be concentrated in the low M values. They can be concentrated in the high M values. Lastly, they can be more or less distributed evenly across M values.

Table 13:

E-M relay error	**CO-9**	**IAC53**
RMSE (sec)	0.04214	0.03473
Max Error (sec)	0.06934	0.15003
RMS % Error	7.33%	2.91%

Table 14:

Static relay error	**BE1B**	**DIAC**	**7SK88**
RMSE (sec)	0.05361	0.02755	0.04632
Max Error (sec)	0.19600	0.03011	0.05084
RMS % Error	6.25%	17.17%	23.83%

Table 15:

Digital relay error	BE1	SPAJ	SR	SEL
RMSE (sec)	0.01059	0.03314	0.03223	0.00807
Max Error (sec)	0.02716	0.10002	0.16283	0.01307
RMS % Error	4.84%	12.82%	3.24%	4.63%

The average RMS error for all the models is 9.22 % .

The time overcurrent relay models can be placed one after the other in increasing phasor frequency with the spaces in between filled through interpolation. The interpolation technique used is the piecewise cubic Hermite polynomial spline interpolation in order to prevent overshoots.

An example of the resulting three dimensional surface of operating time with respect to the multiples of pickup (M) and frequency is shown in Figure 6. Power system overcurrent protection relays are designed to operate at phasor input of 60 Hz with an allowance of around +-0.2 seconds depending on the coordination engineer. Using the model, the range of frequencies that will exhibit operation similar to the operation of at 60 Hz can be determined. This is importance because the frequency of the grid is rarely exactly 60 Hz. However, it is kept to within +- 0.5% of 60 Hz as much as possible.

Figure 6 comes from the models developed for the IAC53, an electromechanical relay. Electromechanical models have a characteristic toppled “L” shape when viewed from the frequency axis. It shows an abnormally high tripping time for low frequencies at low M values (M = 1.5 to 2). The tripping time drops at the region surrounding 60 Hz. The relay is designed to operate at this frequency. Then the tripping time starts a slow rise, giving the semblance of a topple “L”.

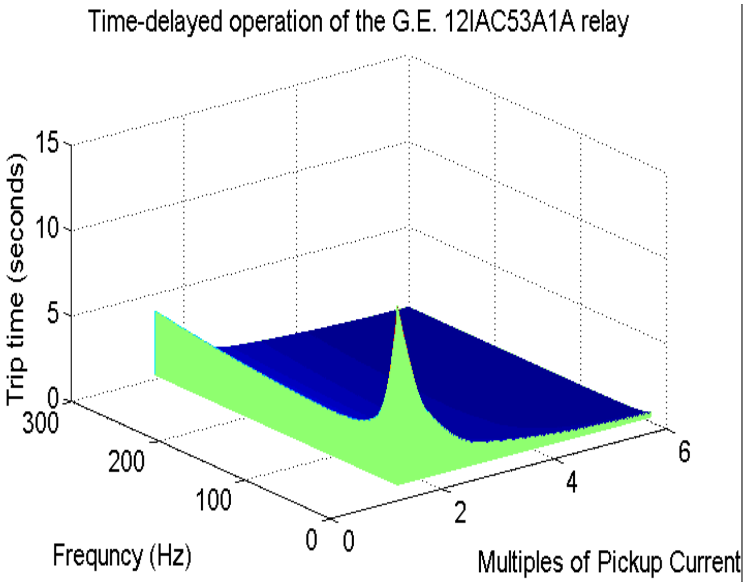

Figure 6: Time-frequency overcurrent characteristic for the IAC53 relay

For a given allowance of 1.5 cycles or 0.025 seconds, the IAC53 relay can reliably be operated within the range of 59-61 Hz. 1.5 cycles is far less than the standard allowance given. It was chosen because fast transients manifest in the range of 3 cycles. The 59-61 Hz range well includes the 59.7-60.3 Hz variation expected for Philippine power systems under normal conditions.

Figure 7 comes from the models developed for the BE1B, a solid state relay. Solid state models have a characteristic flat shape when viewed from the frequency axis. It shows an even performance for the whole range of frequencies characterized in previous research. These devices have bandwidths that can accommodate the largest range of frequencies of inputs without attenuating the inputs. The interpolated solid state relay models reveal that the device will operate as designed for a large input frequency range.

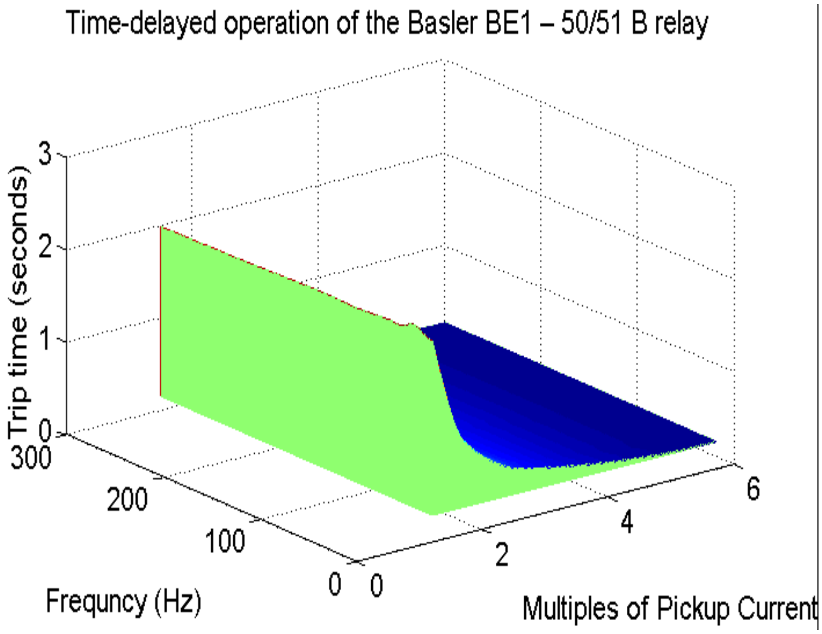

Figure 7: Time-frequency overcurrent characteristic for the BE1B relay

For a given allowance of 1.5 cycles or 0.025 seconds, the BE1B relay can reliably be operated within the range of 20 – 157 Hz. 1.5 cycles is far less than the standard allowance given. I was chosen because fast transients manifest in the range of 3 cycles. The 20 – 157 Hz range well includes the 59.7-60.3 Hz variation expected for Philippine power systems under normal conditions.

Figure 8 comes from the models developed for the BE1, a microprocessor-based relay. Microprocessor-based models have a characteristic spiking at one or both ends when viewed from the frequency axis. This makes the region around 60 Hz appear low and flat. It reveals severe attenuation at low or high frequencies. These devices have bandwidths that are sensitive only to the range of frequencies of inputs near 60 Hz. The interpolated digital relay models reveal that the device filters out edge frequencies and would respond only to the fundamental and frequencies surrounding it.

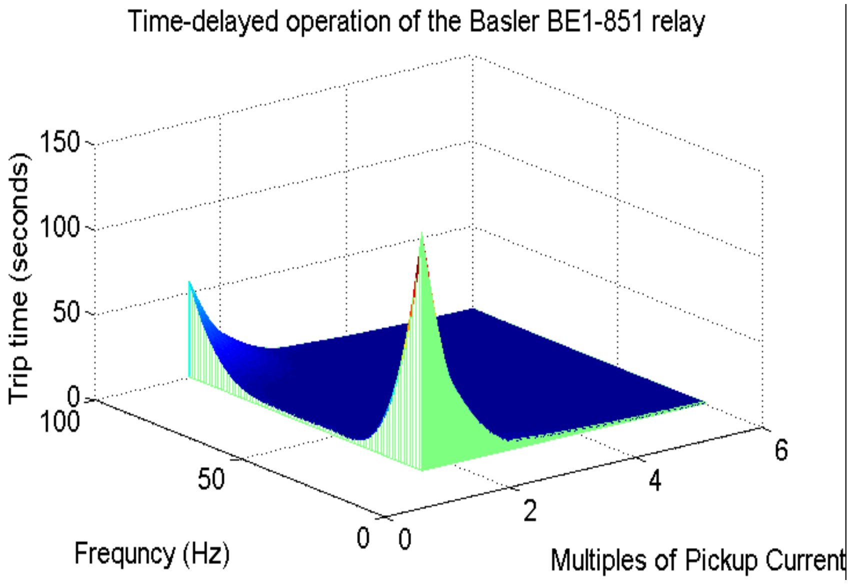

Figure 8: Time-frequency overcurrent characteristic for the BE1B relay

For a given allowance of 1.5 cycles or 0.025 seconds, the BE1 relay can reliably be operated at 60 Hz only. Outside 60 Hz, small attenuation of the signal starts. This relay, like many digital relays bases its operation mainly on the amplitude of the fundamental component of the input.
Table indicates the frequency range for each relay where the predicted operating time differs by a maximum of 1.5 cycles. The maximum range for the data was 10 to 300 Hz.

RELAY	FREQUENCY RANGE
ABB/WH CO-9/Style 264C901A07 (EM)	58 – 62 Hz
G.E. 12IAC53A1A (EM)	59 – 61 Hz
Basler BE1 – 50/51 B (static)	20 – 157 Hz
G.E. DIACA5A (static)	22 – 93 Hz
Siemens 7SK8854 (static)	25 – 300 Hz
Basler BE1-851 (digital)	60 Hz only
ABB SPAJ140C (digital)	59 – 61 Hz
G.E. Multidia SR-760 (digital)	58 – 61 Hz
Schweitzer SEL-587 (digital)	58 – 62 Hz

3.1 Instantaneous Models for Phasor Inputs

Instantaneous operation in relays assume a single value of operating time upon detection of a fault. However, the frequency of the input waveform can cause variations in operating time. The relays characterized have erratic scatter plots of operating time with respect to operating frequency.

The plots do not resemble each other, even when relays of the same generation are compared. In order to model the phenomenon mathematically, high-degree polynomial curves are fitted to regions of data in order to come up with a piecewise closed form approximate but accurate models. A high order polynomial would have the form of (4) . Around three regions are expected to model the relay operation with respect to frequency. A center region would be around 60 Hz, where the relay was designed to operate. The two other regions would be on either side of the region containing 60 Hz to account for the frequencies wherein the device is not expected to operate in. The regression analysis is performed in MATLAB and the boundaries of the regions are set to minimize the least squared error with respect to the data.

$$y(x)=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4+a_5x^5+a_6x^6 \tag{4}$$

The input phasor waveforms are in the form $A\sin(2\pi ft)$. A is the amplitude of the sinusoid. The frequency in Hertz is f. Previous research has experimented on with a frequency range of 10 to 300 Hz. This covers the frequencies around 60 Hz as well as the second to the fifth harmonics. The time in seconds is t. For the purpose of plotting, we have used a time step of $(0.01\div f)$.

The G.E. 12IAC51B1A electromechanical relay was modeled as (5) to give the instantaneous operating time with respect to the phasor frequency of the input. The resulting plot is shown in Figure 9. There is a discontinuity at 40 Hz, which is a boundary between frequency regions in the model. This suggests that numerical techniques may be useful when predicting operating times near boundaries. The RMS error is computed to be 5.6364 milliseconds, which is about a third of a cycle.

$$t_{trip}=\begin{cases} 4.7883\text{e-}005\,x.^3-0.0039675\text{x.}^2+0.098857\text{x.}-0.4933\,, & 10\le x\le 40\\ -1.374\text{e-}012\,x.^7+8.2056\text{e-}010\,x.^6-2.069\text{e-}007\,x.^5+2.8538\text{e-}005\,x.^4 & \\ -0.0023246\text{x.}^3+0.1118\text{x.}^2-2.9397\text{x.}^1+32.649, & 40<x\le 120 \end{cases} \tag{5}$$

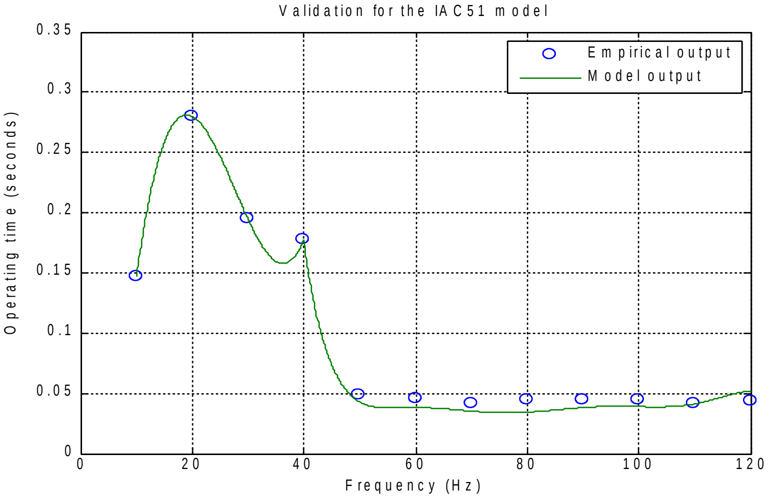

Figure 9: Instantaneous mode frequency model for the IAC51 relay

An equation was developed for each of the electromechanical relays. Equations (6) models the CO-9 relay, another electromechanical relay. The RMSE for the electromechanical relays are listed in Table 16. The average RMSE for electromechanical relays modeled is 1.35E-02 seconds.

$$t_{trip} = \begin{cases} -3.8512\text{e-}006\,x.^{4}+0.00053979\text{x}.^{3}-0.025839\text{x}.^{2}+0.48384\text{x}-2.6157, & 10\le x\le 50 \\ 1.6972\text{e-}011\,x.^{7}-4.9453\text{e-}009\,x.^{6}+5.3599\text{e-}007\,x.^{5}-2.5613\text{e-}005\,x.^{4}+0.00045586\text{x}.^{3}, & 50< x\le 90 \\ 2.0733\text{e-}005\,x.^{3}-0.005898\text{x}.^{2}+0.53973\text{x}.^{1}-15.527, & 90< x\le 120 \end{cases} \tag{6}$$

Model for the CO-9 relay

Table 16: Error values (E-M)

E-M	CO-9	IAC51
RMSE	2.14E-02	5.65E-03

Equations were developed for each of the static analog relays characterized in previous research. Equations (7) , (8) , and (9) model the Basler BE1 – 50/51 B solid state relay , the DIAC solid state relay, and the 7SK88 solid state relay, respectively. The RMSE for the solid state relays are listed in Table 17. The average RMSE for electromechanical relays modeled is 1.60 milliseconds.

$$t_{trip} = \begin{cases} -1.275\text{e-}005\,x.^{3}+0.001098\text{x}.^{2}-0.028285\text{x}.+0.2514, & 10\le x\le 40 \\ 9.1667\text{e-}008\,x.^{4}-2.5917\text{e-}005\,x.^{3}+0.0026828\text{x}.^{2}-0.12042\text{x}.+2.009, & 40< x\le 80 \\ -1.0033\text{e-}006\,x.^{4}+0.00047325\text{x}.^{3}-0.08129\text{x}.^{2}+6.0574\text{x}.^{1}-165.51, & 80< x\le 120 \end{cases} \tag{7}$$

Model for the BE1B relay

$$t_{trip} = \begin{cases} 1.0675\text{e-006}\,x.^{4} - 0.00013355\text{x}.^{3} + 0.0057442\text{x}.^{2} - 0.098875\text{x}.^{1} + 0.6391, & 10 \le x \le 50 \\ 7.6167\text{e-006}\,x.^{3} - 0.0015255\text{x}.^{2} + 0.099523\text{x}.^{1} - 2.0804, & 50 < x \le 80 \\ -0.003484\text{x}.^{2} + 0.63214\text{x}.^{1} - 28.256, & 80 < x \le 100 \\ -0.0022035\text{x}.^{2} + 0.48274\text{x}.^{1} - 26.12, & 100 < x \le 120 \end{cases} \qquad (8)$$

Model for the DIAC relay

$$t_{trip} = \begin{cases} 1.6667\text{e-008}\,x.^{3} + 0.000104\text{x}.^{2} - 0.0052017\text{x}. + 0.0716, & 10 \le x \le 40 \\ -2.3042\text{e-007}\,x.^{4} + 5.6125\text{e-005}\,x.^{3} - 0.005009\text{x}.^{2} + 0.19314\text{x}. - 2.6823, & 40 < x \le 80 \\ -3.0558\text{e-021}\,x.^{4} - 4.8333\text{e-007}\,x.^{3} + 0.000146\text{x}.^{2} - 0.014552\text{x}.^{1} + 0.4867, & 80 < x \le 120 \end{cases} \qquad (9)$$

Model for the 7SK88 relay

Table 17: Error values (Static)

Static	BE1B	DIAC	7SK88
RMSE	4.40E-03	3.24E-04	7.01E-05

Equations were developed for each of the static digital relays characterized in previous research. Equations (10) , (11) , (12), and (13) model the ABB SPAJ140C relay, the BE1-851 relay, the SR-760, and the SEL-587 relay, respectively. The RMSE for the digital relays are listed in Table 17. The average RMSE for the microprocessor-based relays modeled is 0.0706 milliseconds.

$$t_{trip} = \begin{cases} -4.7896\text{e-006}\,x.^{4} + 0.00061699\text{x}.^{3} - 0.027806\text{x}.^{2} + 0.50379\text{x}.^{1} - 2.8035, & 10 \le x \le 50 \\ -2.7171\text{e-012}\,x.^{7} + 5.3198\text{e-010}\,x.^{6} - 3.4325\text{e-008}\,x.^{5} + 7.3553\text{e-007}\,x.^{4,} & 50 < x \le 80 \\ -1.0979\text{e-006}\,x.^{4} + 0.00043214\text{x}.^{3} - 0.063254\text{x}.^{2} + 4.0797\text{x}.^{1} - 97.712, & 80 < x \le 120 \end{cases} \qquad (10)$$

Model for the SPAJ relay

$$t_{trip} = \begin{cases} -1.2992\text{e-008}\,x.^{5} + 1.3396\text{e-006}\,x.^{4} - 4.4471\text{e-005}\,x.^{3} + 0.00050854\text{x}.^{2}, & 20 \le x \le 50 \\ -8.2027\text{e-011}\,x.^{6} + 1.6029\text{e-008}\,x.^{5} - 1.0303\text{e-006}\,x.^{4} + 2.1894\text{e-005}\,x.^{3,} & 50 < x \le 80 \\ 2.2831\text{e-019}\,x.^{10} - 3.9937\text{e-017}\,x.^{9} + 1.7512\text{e-015}\,x.^{8}, & 80 < x \le 100 \end{cases} \qquad (11)$$

Model for the BE1-851 relay

$$t_{trip} = \begin{cases} -0.001114\text{x}.^{2} + 0.06681\text{x}.^{1} - 0.8316, & 20 \le x \le 40 \\ 2.7833\text{e-006}\,x.^{3} - 0.00050493\text{x}.^{2} + 0.028998\text{x}.^{1} - 0.47236, & 40 < x \le 80 \\ -0.0031785\text{x}.^{2} + 0.57187\text{x}.^{1} - 25.366, & 80 < x \le 100 \end{cases} \qquad (12)$$

Model for the SR-760 relay

$$t_{trip} = \begin{cases} 0.001188\text{x.}^{2} - 0.09962\text{x.}^{1} + 2.105, & 30 \le x \le 50 \\ -2.21\text{e-}005\, x.^{3} + 0.0044425\text{x.}^{2} - 0.29492\text{x.}^{1} + 6.496, & 50 < x \le 80 \\ -2.6\text{e-}005\, x.^{2} + 0.00507\text{x.}^{1} - 0.2196, & 80 < x \le 100 \end{cases} \tag{13}$$

Model for the SEL-587 relay

Table 18: Error values (Digital)

Digital	SPAJ	BE1	SR	SEL
RMSE	6.44E-04	2.37E-04	1.73E-03	2.16E-04

The average RMSE for all the instantaneous relay models is 3.86 milliseconds with a standard deviation of 6.90 milliseconds.

Instantaneous overcurrent relays operate as fast as it could once the minimum current is reached. The value of the minimum current is set for a phasor frequency of 60 Hz. If the input is of another frequency, the operating time changes as given by the models in the preceding section. The current needed to activate the relay operation also varies with the frequency of the input. Color-coded density diagrams were developed to show relay operating time with respect to the input frequency and the current amplitude necessary to trip the relay at that frequency. Cubic Hermite spline interpolation was used to determine current amplitudes in between those tested in previous research.

The density diagram for the G.E. 12IAC51B1A electromechanical relay is shown in Figure 10. The color of the diagram correspond to the time it would take for the relay to trip give a certain frequency and current magnitude. The dark blue area correspond to negative time, that is, a region where the relay shall not trip. The light blue area is where relay operation is fast and thus, optimal. The model reveals that electromechanical relays require an almost linear increase in current magnitude as the frequency of the input increases for the relay to operate. There is therefore risk of damaging this type of relay when given high frequency inputs, as the relay will not operate until a high value of current is sensed. Variations in phasor frequency can cause the electromechanical relay to trip 4-5 times longer than what it was designed to.

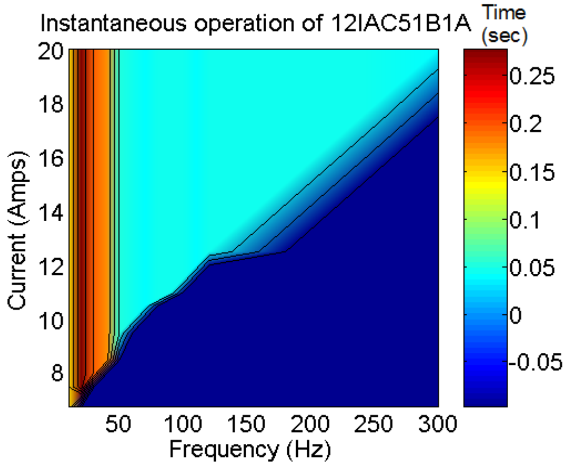

Figure 10: Instantaneous time density diagram for the IAC51 relay

The density diagram for the Basler BE1-50/51 B solid state relay is shown in Figure 11. The color of the diagram correspond to the time it would take for the relay to trip give a certain frequency and current magnitude. The dark blue area correspond to negative time, that is, a region where the relay shall not trip. The light blue area is where relay operation is fast and thus, optimal. The model reveals that solid state relays require an almost constant (flat) current magnitude necessary as the frequency of the input increases for the relay to operate. Therefore there is little variation in the current needed to trip this type of relay even when input phasor frequencies are varied. Variations in phasor frequency can cause static analog relays to trip 3-5 times longer than what it was designed to.

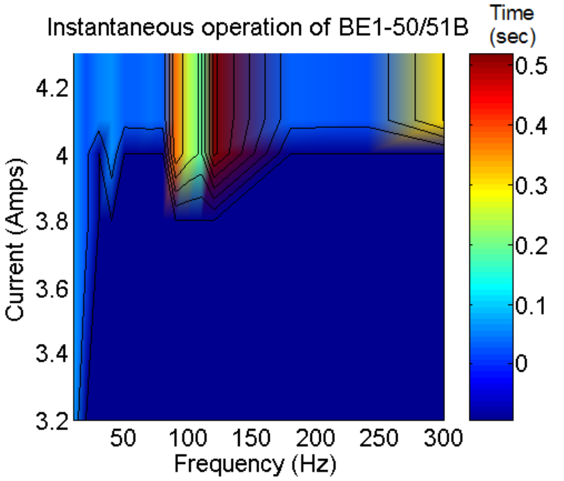

Figure 11: Instantaneous time density diagram for the BE1B relay

The density diagram for the G.E. MULTILIN SR-760 microprocessor-based relay is shown in Figure 12. The color of the diagram correspond to the time it would take for the relay to trip give a certain frequency and current magnitude. The dark blue area correspond to negative time, that is, a region where the relay shall not trip. The light blue area is where relay operation is fast and thus, optimal. The model reveals that digital relays require frequencies of a significantly smaller range for the relay to operate. This is due to the filtering algorithms implemented in the microprocessor. If the input frequency is outside the bandwidth of the relay, the current input will be attenuated to the point that the relay will not operate even at relatively high currents and long times. Therefore, static digital relay operation is expected to be closer to the designed response at 60 Hz if the relay does operate. Still, variations in phasor frequency can cause static analog relays to trip 3 times longer than what it was designed to.

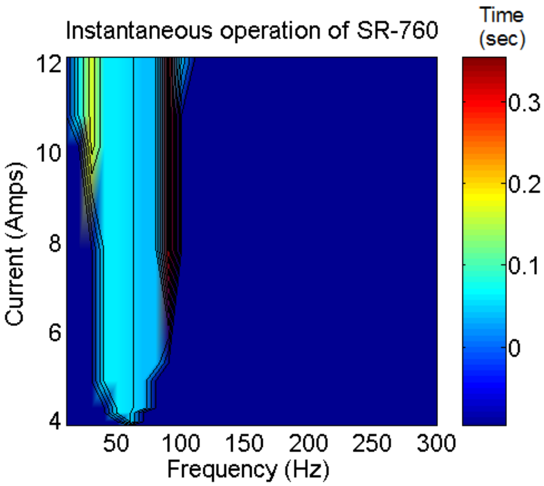

Figure 12: Instantaneous time density diagram for the BE1B relay

The density diagrams reveal that relay instantaneous tripping time for any relay generation type can take 3-5 times longer if the frequency of the input phasor were varied. The current magnitude required to trip the relay can also increase to more than twice the designed value. Operation at certain frequencies may thus cause relay damage due to higher currents and longer tripping times.

IV. Conclusion and Recommendations

4.1 Conclusion

Mathematical models were developed for relay response time with respect to frequency and current magnitude. Seventy-seven time-delayed models were created. Nine models were developed with respect to frequency.

Model accuracy was verified against data from previous research for all the models. The average root mean square error for the time overcurrent models is 8.85 milliseconds. The average RMSE for the instantaneous models is 3.86 milliseconds. The relay time overcurrent models at 60 Hz were verified against relay manufacturers' very inverse curves with an average RMSE of 0.032 seconds.

From the time overcurrent models, further conclusions arise:

a) Electromechanical time overcurrent models show a very high tripping time for low frequencies at low multiple of pickup current values (M=1.5 to 2).

b) Solid state relays models show consistent performance for the entire frequency range modeled.

c) Microprocessor-based models reveal a narrow frequency range of operation centered at 60 Hz. Outside of the range, severe attenuation of the input is observed.

d) Through interpolation, the range of frequencies for which the operation of the relays is as designed, or is comparable to relay operation of 60 Hz, can be found for any prescribed error allowance.

From the instantaneous relay models, further conclusions arise:

a) Electromechanical models reveal that electromechanical relays require an almost linear increase in current magnitude as the frequency of the input increases for the relay to operate.

b) Solid state models reveal that solid state relays require an almost constant (flat) current magnitude necessary as the frequency of the input increases for the relay to operate.

c) Microprocessor-based models show that digital relays require frequencies to be of a significantly smaller range for the relay to operate.

d) The instantaneous models reveal that relay instantaneous tripping time for any relay generation type can take 3-5 times longer if the frequency of the input phasor were varied. The current magnitude required to trip the relay can also increase to more than twice the designed value.

Dynamic relay modeling was found to be very useful particularly for the following:

a) Planning to upgrade or update protective relays. The models will serve to check the compatibility of the candidate relay with the particular transients occurring in the particular power system.

b) Fine-tuning of relay settings to come up with the optimal choice of settings based on power system operating characteristics. The traditional approach of using “rule of thumb” in setting relays to compensate for inrush currents during system energizing can now be improved since operating points can be mapped for various frequency and current ranges using relay models

4.2 Recommendations

Dynamic relay modeling offers insight into the random behavior of relays. However, relay dynamic modeling is normally complicated, tedious, and time-consuming. Basing the dynamic models on the results of relay characterization allows a uniform procedure of modeling overcurrent relays of different generation types. This also makes the models highly dependent on the data from the characterization. Obtaining more data will better validate and improve the models developed.

Furthermore, power system simulations normally include fault conditions with multi-frequency transients. Models for these could be developed.

V. References

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[2] Yujie Zhang et al, *Modeling and Testing of Protection Devices for SPS using MATLAB/Simulink and VTB*, IEEE, 2007

[3] Mietek Glinkowski, Jules Esztergalyos, *Transient Modeling of Electromechanical Relays Part 1 , IEEE Transactions on Power Delivery*, Vol. 11, No.2, April 1996

[4] Florentino A. Tempra, *Characterization of Overcurrent Relays using Open-loop Transient-based Test Method*, Master of Science in Electrical Engineering Thesis, University of the Philippines, Diliman, 2004

VI. Appendix A: Relay List

6.1 Electromechanical Relays

Instead of having multiple settings (e.g. inverse curves, instantaneous or time overcurrent modes), electromechanical relays have groups of similarly constructed relays with a separate relay corresponding to a different relay setting.

1. G.E. 12IAC53A1A, G.E. 12IAC51B1A, G.E. 12IFC53A1A
2. A.B.B./W.H. CO-9/Style 264C901A07, W.H. CO-9H111N/Style 264C901A07

6.2 Solid State (Static Analog) Relays

3. Basler BE1 – 50/51 B
4. G.E. DIACA5A
5. Siemens 7SK8854

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6.3 Microprocessor-based (Static Digital) Relays]

6. A.B.B SPAJ140C
7. Basler BE1-851
8. G.E. Multidia SR-760
9. Schweitzer SEL-587